ENTROPY CLOSURES IN UNCERTAINTY QUANTIFICATION

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Abstract. Using polynomial closures in Uncertainty Quantification for hyperbolic problems can lead to non-hyperbolic moment systems as well as oscillatory solutions. The Intrusive Polynomial Moment (IPM) method [1] uses an entropy reconstruction to guarantee hyperbolicity of the resulting moment system while mitigating oscillations in the case of scalar equations. One of the main challenges facing the IPM method is the need to repeatedly solve an optimization problem in order to perform the entropy reconstruction. Consequently, the increased numerical costs make it hard for IPM to compete with classical, non-intrusive methods such as Stochastic Collocation.

In this talk, we present new techniques to accelerate the IPM method while exploiting its intrusive nature to obtain an increased accuracy for steady problems: When using pseudo-time steps to iterate the moments to a steady state, we propose to not fully converge the optimization problem. In fact, we perform only a single Newton iteration towards the exact minimizer, i.e. we simultaneously iterates both, the dual variables of the optimization problem as well as the moments to a steady state. Consequently, the numerical costs of performing the entropy reconstruction are significantly decreased. Furthermore, we propose to perform most of the pseudo-time iterations with an inaccurate but cheap method (in our case Stochastic Collocation) and only use the IPM method for the last iterations. Thereby, the overall computational costs are dominated by the large amount of Collocation iterations and not by the costs to obtain an accurate IPM solution. Additionally, we make use of the intrusive nature of IPM by locally refining the stochastic space in regions with non-smooth solutions.

We demonstrate the effectiveness of the different approaches by investigating an inviscid flow around a NACA0012 profile with an uncertain angle of attack.

References

[1] Poëtte, Gaël and Després, Bruno and Lucor, Didier. Uncertainty quantification for systems of conservation laws. Elsevier *Journal of Computational Physics*, 228(7):2443–2467, 2009.

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